



# Introducing relations between activities and goods consumption in microeconomic time use models



Sergio R. Jara-Díaz<sup>a,\*</sup>, Sebastian Astroza<sup>b,1</sup>, Chandra R. Bhat<sup>c,1</sup>, Marisol Castro<sup>a,2</sup>

<sup>a</sup> Universidad de Chile, Transport System Division, Casilla 228-3, Santiago, Chile

<sup>b</sup> The University of Texas at Austin, Department of Civil, Architectural and Environmental Engineering, 301 E. Dean Keeton St. Stop C1761, Austin TX 78712-1172, U.S.A

<sup>c</sup> The University of Texas at Austin, Department of Civil, Architectural and Environmental Engineering, 1 University Station C1761, Austin, TX 78712-0278, U.S.A

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## ABSTRACT

We present a microeconomic model for time use and consumption for workers with an improved treatment of the (technical) relations between goods and time. In addition to the traditional time and income constraints, an improved set of restrictions involving explicit relations between consumption of goods and time assigned to activities is included in two versions. In each version, a system of equations involving a subset of the consumer's decision variables is obtained, including (1) work time, (2) activities that are assigned more time than the minimum, and (3) goods that are consumed above the minimum. The system cannot be solved explicitly in the endogenous decision variables but is used to set a stochastic system for econometric estimation through maximum likelihood. The models are applied to analyze weekly time use and consumption data from Netherlands for year 2012. The results obtained by this new “goods and time” framework are compared with previous research in terms of the value of leisure and the value of work, showing substantial differences in the valuation of time.

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## 1. Introduction

Workers' behavior in terms of their use of time has been studied from many perspectives and in many disciplines, including labor economics and transportation. Among such studies, the common thread has been the attempt to explain workers' time use as a function of exogenous variables with the aim to understand the frequency, duration, and sequence of activity participations (see Bhat and Koppelman, 1993). A key component of such a time-use analysis is an understanding of workers' willingness to pay to decrease travel time, which incorporates several effects, including the value of doing something else (leisure or work). In particular, changes in transportation affect travel time and, therefore, have an impact on the allocation of time to non-travel activities.

Many approaches have been used to understand the allocation and valuation of time. One of the most popular approaches is the expansion of the basic microeconomic consumer theory by including time in a utility function that represents

\* Corresponding author. Fax: (562)6894206.

E-mail addresses: [jaradiaz@ing.uchile.cl](mailto:jaradiaz@ing.uchile.cl) (S.R. Jara-Díaz), [sastroza@utexas.edu](mailto:sastroza@utexas.edu) (S. Astroza), [bhat@mail.utexas.edu](mailto:bhat@mail.utexas.edu) (C.R. Bhat), [marisolcastro@gmail.com](mailto:marisolcastro@gmail.com) (M. Castro).

<sup>1</sup> Fax: 512 475 8744.

<sup>2</sup> Fax: (562)6894206.

unconstrained ordinal preferences and adding temporal restrictions besides the budget constraint. As known, consumer theory looks at the individual as if he or she chooses what he or she prefers; from this viewpoint, utility (an unobservable artifact) is only a construction from which (observable) demand functions can be obtained. The essence of these models is that the individual assigns money to buy goods and invests time to undertake activities through a strategic underlying equilibrium mechanism between money and time; as known, time cannot be “saved” but it can certainly be reallocated after changes in exogenous conditions (e.g. income, prices). Since these microeconomic models simultaneously consider time and income constraints and choices involving money and time, different types of time values can be developed, including value of time as a resource, value of working time, and value of assigning time to an activity. These values are important in the evaluation of transportation policies, because the benefits of the reduction of travel time can be economically measured using the different estimated values of time.

Becker's study (1965) appears to be the first to include time and its value in microeconomic consumer theory. Becker proposed final goods—combinations of market goods and preparation time—as the argument of the utility function and the inclusion of a total time constraint, time equivalent of the typical total income restriction (see Pollack, 2003 and Cherchye et al., 2015 for discussions). According to Becker's framework, the value of time as a resource is equal to the individual's wage rate. Some years later, DeSerpa (1971) modified Becker's model by including directly goods consumption and time allocation in the utility function. DeSerpa also added technological restrictions, linking the consumption of goods with a certain minimum time of consumption. DeSerpa was the first to clearly define leisure activities (those the individual assigns more time than the minimum) and its value, obtaining relations between the different values of time. As a derivation of the first order conditions, DeSerpa indicated that the value of leisure is equal to the total value of work (wage rate plus the intrinsic value of working time); he further indicated that the willingness to pay to save time in an activity is equal to the value of leisure (the value of doing something else) plus the value of time assigned to that activity. Evans (1972) proposed a utility function depending only on time assigned to activities and a new type of restriction linking time assigned to different activities, i.e., the time assigned to a particular activity could be directly related to the time assigned to another one. As noted by Jara-Díaz (2003), the money budget constraint in Evans' model contains a transformation of activities into the consumption of goods that can be interpreted as another type of technical relation between goods and time.

Since the theoretical frameworks of Becker, DeSerpa, and Evans, the literature of microeconomic time use models has expanded in several directions (for a detailed review, see Jara-Díaz, 2007), including the study of travel time and mode choice within the goods-leisure tradeoff framework (Train and McFadden, 1978), investigations related to home-production (Gronau, 1986), time-specific analysis (Pawlak et al., 2015, López-Ospina et al., 2015) and of course more theoretical developments regarding the type of restrictions and variables that should be considered in the consumer theory framework. Thus, building from DeSerpa (1971) and Evans (1972), Jara-Díaz (2003) showed that there are two types of *technical* relations between goods consumed and time assigned to activities. Simply put, they can be stated as minimum activity times that depend on the amount of goods needed to perform them (a generalization of DeSerpa) and minimum consumption of goods induced by the activities undertaken (a generalization of Evans). These two families of relations can be treated as yet additional constraints in a consumer behavior microeconomic framework including time use, such that exogenous changes (e.g. re-design of the transit system or improvements in communication systems) will affect these relations and induce a change in time use patterns.

If a good is consumed, there may be a minimum consumption level or expenditure associated with that good. Similarly, if an activity type is participated in, there may be a minimum level of time investment required in the participation (for example, taking a child to the doctor's office entails some minimum level of time spent at the doctor's office). Individuals may generally prefer to strictly stick to the minimum consumption (or expenditure) level for some goods (let this set of goods be denoted by  $G^R$ ), while may consume (or expend) more than the minimum for some other goods (let this set of goods be denoted by  $G^F$ ). In a similar vein, individuals may invest the minimum possible time for certain activity types (let this set of activity types be  $A^R$ ), while they may invest more than the minimum for certain activity types (let this set of activity types be  $A^F$ , the leisure activities according to DeSerpa).

In their simplest form, both types of technical relations were introduced by Jara-Díaz and Guevara (2003) and expanded in Jara-Díaz et al. (2008) as exogenously given minimum levels of good consumption and time allocation, a very simplified manner to account for these types of constraints. Jara-Díaz et al.'s (2008) formulation considered, as usual, that consumption of different goods and time assignment to different types of activities are the consumer's decision variables. Although quite limited as a representation of the technical constraints, the simple formulation allowed for a closed analytical solution in three types of variables: (1) time assigned to activities beyond the minimum (those in  $A^F$ ), (2) work time, and (3) amount of goods consumed above the corresponding minimum (those in  $G^F$ ). By considering additive interdependent errors in the resulting equation system, the utility parameters can be estimated and, for the first time, the (marginal) values of leisure and work were actually estimated and computed. Here, the value of leisure is equal to the value of time as a resource.

However, there is a component of the total value of leisure that is different from the value of time as a resource. This difference cannot be revealed with Jara-Díaz's (2008) model because, as suggested by Konduri et al. (2011) and shown by Jara-Díaz and Astroza (2013), explicit relations between goods consumed and time assigned are needed. To begin accounting for this, here we consider two models: one where time allocated to activities impose minimum consumption of certain goods, a generalization of Evans (1972); and another where goods consumed impose a minimum necessary time to activities, a generalization of DeSerpa (1971). Unlike previous empirical models, all these minima become endogenous. That is, we explicitly tie goods consumption (or expenditures) levels to time-use. Although closed solutions cannot be obtained in either

case, we show that stochastic specifications can be formulated and estimated in both cases. Unfortunately, this cannot be done when both type of constraints are simultaneously introduced.

To our knowledge, this is the first time such a set of relationships is included in the time use model formulation. Indeed, while there have been important recent developments in time-use modeling (including Bhat's (2008) and Castro et al. (2012) multiple discrete-continuous choice model, Jara-Díaz et al.'s (2008) micro-economic model, and the use of a structural equations model by Konduri et al., 2011 and Dane et al., 2014), all of these efforts recognize that a better treatment of the (technical) relations between goods consumed and time use is a critical need.

The proposed model is applied to weekly time use and consumption data obtained from the 2012 LISS (Longitudinal Internet Studies for the Social Sciences) panel. This panel is administered by CentERdata ([www.lissdata.nl](http://www.lissdata.nl)) and is representative of the Dutch population. The LISS panel is a standard social survey, to which a questionnaire was added to gather information about time use and consumption (Cherchye et al., 2012). Obtaining data on both time use and goods consumption from the same source is not common and previous works have needed to develop a methodology to merge time use surveys and consumer expenditure data (see, for example, the imputation of income and expenses performed by Olguín, 2008, and the merging of the 2008 American Time Use Survey and the 2008 Consumer Expenditure Survey by Konduri et al., 2011). To our knowledge, the LISS panel is one of the few surveys in the world that captures both time allocation and goods consumption information. Previous studies (Colella and van Soest, 2013; Rubin, 2015) have used the LISS data to explore the association between time use, time constraints and consumption, but this is the first study that uses the data to understand the link between these variables.

The remainder of the paper is structured as follows. In the next section we formulate the two versions of the micro-economic model. Section 3 contains the stochastic counterpart and presents the maximum likelihood estimation procedure. Section 4 describes the data, while Section 5 discusses the empirical results. The final section summarizes the approach and results, and identifies future research directions.

## 2. Model formulation

### 2.1. The common elements

Consider the following time use – goods consumption model for workers:

$$\text{Max } U(\mathbf{T}, \mathbf{X}) = \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j X_j^{\varphi_j} \quad (1)$$

$$\text{s.t. } wT_w + I - \sum_j P_j X_j - c_f \geq 0 \quad (\lambda) \quad (2)$$

$$\tau - T_w - \sum_i T_i = 0 \quad (\mu) \quad (3)$$

In Eq. (1),  $U$  is a Cobb-Douglas utility function that depends on time allocation ( $T$ ) and good consumption ( $X$ ). The time allocation vector  $\mathbf{T}$  includes the time assigned to work  $T_w$  and the time  $T_i$  assigned to each non-work activity  $i$  during time period. The good consumption vector  $X$  contains the consumption level  $X_j$  ( $j=1,2,\dots,J$ ) for each good  $j$ , consumed during the same time period. The parameters of the utility function are a positive constant  $\Omega$ , the time parameters  $\theta_w$  and  $\theta_i$  for all  $i$ , and the consumption parameters  $\varphi_j$  for all  $j$ . Note that  $\theta_i$ ,  $\theta_w$  and  $\varphi_j$  represent the elasticity of the utility with respect to time assigned to activity  $i$ , time assigned to work, and consumption of good  $j$  respectively. These elasticities measure the responsiveness of utility to a marginal change in levels of good consumption or time assigned to activities (*ceteris paribus*). For example, if  $\theta_w=0.20$ , a 1% increase in working time would lead to a 0.20% increase in utility.

The first constraint (Eq. (2)) is the income constraint that accounts for all expenses and all types of income.  $w$  is the wage rate,  $I$  is the income obtained from non-work activities (such as pensions, gifts and investment returns),  $P_j$  is the unitary price of good  $j$  and  $c_f$  represents the total fixed expenditures (those that do not depend on the goods or services purchased in the period). The second constraint (Eq. (3)) is the total time constraint for activity times. The Lagrange multipliers  $\lambda$  and  $\mu$  represent the marginal utility of increasing available money and increasing available time, respectively.

The novelty in this paper is the family of technological constraints. In addition to the income constraint and the total time constraint, we include constraints that impose minimum consumption of goods and minimum allocation of time. We propose two different versions of our model: a) one model with exogenous minimum time allocations and endogenous minimum good consumptions (generalizing Evans, 1972), and b) another model with endogenous minimum time allocations (generalizing DeSerpa, 1971) and exogenous minimum good consumptions.

### 2.2. Model with endogenous minimum consumption of goods and exogenous minimum time allocations

In addition to (1)–(3) we propose the inclusion of the following family of constraints:

$$T_i - T_i^{\min} \geq 0 \quad \forall i \quad (\kappa_i) \quad (4)$$

$$X_j - \sum_i \alpha_{ij} T_i \geq 0 \quad \forall j \quad (\psi_j) \quad (5)$$

The first technological constraint (Eq. (4)) incorporates in the model the existence of minimum time allocations for each activity  $i$ , represented by  $T_i^{\min}$  ( $T_i^{\min}$  can be zero for certain activities). The second technological constraint (Eq. (5)) represents the minimum consumption of a good that is needed when a certain activity is undertaken, with  $\alpha_{ij}$  representing the amount of good  $j$  needed to perform activity  $i$  (per unit time). The Lagrange multipliers  $\kappa_i$  and  $\psi_j$  represent the marginal utility of reducing the minimum time for activity  $i$  and reducing the minimum consumption of good  $j$ , respectively. Note that this type of relation is like an aggregated generalization of the implicit set of constraints in Evans' model (1972) that turns time use into goods consumption through a matrix  $Q$  (see Jara-Díaz, 2003 for a detailed discussion).

The First Order Conditions (F.O.C.) with respect to the decision variables  $T_i$ ,  $T_w$  and  $X_j$  may be derived in a straightforward fashion as shown in Appendix A. These conditions are:

$$\frac{\theta_i U}{T_i} + \kappa_i - \mu - \sum_j \psi_j \alpha_{ij} = 0 \quad [\text{for decision variable } T_i] \quad (6)$$

$$\frac{\theta_w U}{T_w} + \lambda w - \mu = 0 \quad [\text{for decision variable } T_w] \quad (7)$$

$$\frac{\varphi_j U}{X_j} + \psi_j - \lambda P_j = 0. \quad [\text{for decision variable } X_j] \quad (8)$$

The F.O.C.'s above have an intuitive interpretation. According to Eq. (6), activities that are assigned more than the minimum time necessary ( $\kappa_i = 0$ ) and do not impose a minimum level of consumption on any of the goods, have the same marginal utility, following a common result in time use models since DeSerpa (1971), who was the first one to propose that all the freely chosen activities (activities that are assigned more time than necessary, activities that DeSerpa called "leisure activities") have the same marginal utility. Of course, the special case is work (see Eq. (7)); the marginal utility of time assigned to work plus the wage rate- which is multiplied by the marginal utility of money- has to be equal to the marginal utility of the activities that are assigned more than the minimum necessary. In other words, the total value of work has to be equal to the value of leisure time, as defined by DeSerpa (1971). For activities that are assigned more than the minimum necessary ( $\kappa_i \neq 0$ ) and do not impose minimum consumption for any of the goods, the marginal utility of the time assigned to the activity plus the marginal utility of a marginal relaxation of the minimum constraint has to be equal to the marginal utility of the freely chosen activities, as can be seen in Eq. (6). In the particular case that one of the activities impose certain minimum good consumption, an extra term has to be added in the equilibrium:  $\sum_j \psi_j \alpha_{ij}$ . This additional term represents the impact on utility of the change on the consumption structure when the time assigned to the specific activity is marginally increased. Finally, according to Eq. (8), for those goods with a level of consumption greater than the minimum necessary ( $\psi_j = 0$ ), the price-normalized marginal utility of good has to be equal to the marginal utility of money. For those goods that are consumed only the minimum necessary, the price-normalized marginal utility of good plus the marginal utility of a relaxation of the minimum consumption constraint has to be equal to the marginal utility of money.

The F.O.C. with respect to  $X_k$  (Eq. (8)) for  $k \in G^F$  (i.e.  $\psi_k = 0$ ) is:

$$\frac{\varphi_k U}{X_k} - \lambda P_k = 0. \quad (9)$$

Adding (9) over  $G^F$  and defining  $\varphi \equiv \sum_{k \in G^F} \varphi_k$  we get:

$$\frac{\lambda}{U} = \frac{\varphi}{\sum_{k \in G^F} P_k X_k}. \quad (10)$$

imposing the budget constraint, we can rewrite the denominator of the right side of Eq. (10) and get:

$$\frac{\lambda}{U} = \frac{\varphi}{(wT_w + I - c_f - \sum_{j \in G^R} P_j X_j)}. \quad (11)$$

Recalling that for  $j \in G^R$ ,  $X_j = \sum_i \alpha_{ij} T_i$  and noting that if we define  $\gamma_i = \sum_{j \in G^R} P_j \alpha_{ij}$  then we can write  $\sum_{j \in G^R} P_j X_j = \sum_i \gamma_i T_i$ . Given that the summation in the denominator of the right side of Eq. (11) can be split into two parts, we can write the following:

$$I - c_f - \sum_{j \in G^R} P_j X_j = I - c_f - \sum_{h \in A^F} \gamma_h T_h - \sum_{i \in A^R} \gamma_i T_i. \quad (12)$$

As for  $i \in A^R$   $T_i = T_i^{min}$ , there are three terms in the right side of Eq. (12) that are fixed. Recalling that the sum of these three terms is defined by Jara-Díaz et al. (2008) as committed expenses  $E_c$ , then Eq. (11) can be re-written as:

$$\frac{\lambda}{U} = \frac{\varphi}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)}. \quad (13)$$

Dividing Eq. (9) by  $U$  and replacing (13) we obtain the first equation in our system for  $k \in G^F$ :

$$P_k X_k = \frac{\varphi_k}{\varphi} \left( wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h \right). \quad (14)$$

Now consider the F.O.C. for  $T_h$  (Eq. (6)) with  $h \in A^F$  (i.e.  $\kappa_h = 0$ ), which is:

$$\frac{\theta_h U}{T_h} - \mu - \sum_j \psi_j \alpha_{hj} = 0. \quad (15)$$

Adding Eq. (15) over  $A^F$  and defining  $\theta \equiv \sum_{h \in A^F} \theta_h$  we get:

$$\frac{\mu}{U} = \frac{\left( \theta - \frac{\sum_{h \in A^F} \sum_j \psi_j \alpha_{hj} T_h}{U} \right)}{\sum_{h \in A^F} T_h}. \quad (16)$$

We can solve Eq. (8) for  $\psi_j$ :

$$\psi_j = \lambda P_j - \varphi_j U / X_j \quad (17)$$

Replacing (17) and (13) in (16):

$$\frac{\mu}{U} = \frac{\left( \theta - \frac{\varphi \sum_{h \in A^F} \gamma_h T_h}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} + \sum_{h \in A^F} \sum_j \frac{\varphi_j \alpha_{hj} P_j T_h}{P_j X_j} \right)}{\sum_{h \in A^F} T_h}. \quad (18)$$

Rewriting the denominator of Eq. (18) based on the total time constraint and defining committed time as  $T_c \equiv \sum_{i \in A^R} T_i$ :

$$\frac{\mu}{U} = \frac{\left( \theta - \frac{\varphi \sum_{h \in A^F} \gamma_h T_h}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} + \sum_{h \in A^F} \sum_j \frac{\varphi_j \alpha_{hj} P_j T_h}{P_j X_j} \right)}{(\tau - T_w - T_c)}. \quad (19)$$

Dividing Eq. (15) by  $U$  and replacing (19) we obtain the second equation in our system for  $i \in A^F$ :

$$\frac{\theta_i}{T_i} - \frac{\left( \theta - \frac{\varphi \sum_{h \in A^F} \gamma_h T_h}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} - \sum_{h \in A^F} \sum_j \frac{\varphi_j \alpha_{hj} P_j T_h}{P_j X_j} \right)}{(\tau - T_w - T_c)} - \frac{\varphi \gamma_i T_i}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} + \sum_j \frac{\varphi_j \alpha_{ij} P_j T_i}{P_j X_j} = 0 \quad (20)$$

Dividing (7) by  $U$  and replacing (13) and (19) we get the third equation of our system:

$$\frac{\theta_w}{T_w} + \frac{\varphi w}{(wT_w - E_c - \sum_{i \in A^F} \gamma_i T_i)} - \frac{\left( \theta - \frac{\varphi \sum_{h \in A^F} \gamma_h T_h}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} + \sum_{h \in A^F} \sum_j \frac{\varphi_j \alpha_{hj} P_j T_h}{P_j X_j} \right)}{(\tau - T_w - T_c)} = 0. \quad (21)$$

Eqs. (14), (20) and (21) form a system of  $|A^F| + |G^F| + 1$  equations with the same number of unknowns. These unknown decision variables are work time, time assigned to those activities that do not stick to the exogenous minimum, and amount of goods consumed above the corresponding minimum.

Once the system is solved, the rest of the variables (goods and time) can be found as:

$$T_i = T_i^{min} \quad \forall i \in A^R \quad (22)$$

$$X_j = \sum_i \alpha_{ij} T_i \quad \forall j \in G^R \quad (23)$$

The value of time as a resource, or value of leisure, can be obtained as:

$$\frac{\mu}{\lambda} = \frac{\left( \theta - \frac{\varphi \sum_{h \in A^F} \gamma_h T_h}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} + \sum_{h \in A^F} \sum_j \frac{\varphi_j \alpha_{hj} P_j T_h}{P_j X_j} \right) (wT_w - E_c - \sum_{i \in A^F} \gamma_i T_i)}{\varphi (\tau - T_w - T_c)}, \quad (24)$$

and then the value of work can be obtained from Eq. (7):

$$\frac{\partial U / \partial T_w}{\lambda} = \frac{\mu}{\lambda} - w. \quad (25)$$

### 2.3. Model with exogenous minimum consumption of goods and endogenous minimum time allocations

As an alternative model, in addition to (1)–(3) and instead of (4) and (5), we propose the inclusion of the following family of constraints:

$$T_i - \sum_j \beta_{ij} X_j \geq 0 \quad \forall i \quad (\kappa_i) \quad (26)$$

$$X_j - X_j^{\min} \geq 0 \quad \forall j \quad (\psi_j) \quad (27)$$

The first technological constraint (Eq. (26)) represents the existence of minimum time allocations that are needed when a certain good is consumed, with  $\beta_{ij}$  representing the amount of time needed to be invested in activity  $i$  per unit of consumption of good  $j$ . The second technological constraint (Eq. (27)) incorporates in the model the existence of minimum consumption of goods for each good  $j$ , represented by  $X_j^{\min}$  ( $X_j^{\min}$  can be zero for certain goods).

The First Order Conditions (F.O.C.) with respect to the decision variables  $T_i$ ,  $T_w$  and  $X_i$  may be derived in a straightforward fashion as shown in Appendix B. These conditions are:

$$\frac{\theta_i U}{T_i} - \mu + \kappa_i = 0 \quad [\text{for decision variable } T_i] \quad (28)$$

$$\frac{\theta_w U}{T_w} + \lambda w - \mu = 0 \quad [\text{for decision variable } T_w] \quad (29)$$

$$\frac{\varphi_j U}{X_j} - \lambda P_j + \psi_j - \sum_i \beta_{ij} \kappa_i = 0. \quad [\text{for decision variable } X_j] \quad (30)$$

The F.O.C. with respect to  $T_h$  (Eq. (28)) for  $h \in A^F$  (i.e.  $\kappa_h = 0$ ) is:

$$\frac{\theta_h U}{T_h} - \mu = 0. \quad (31)$$

Adding (31) over  $A^F$  and recalling that  $\theta \equiv \sum_{h \in A^F} \theta_h$  we get:

$$\frac{\mu}{U} = \frac{\theta}{\sum_{h \in A^F} T_h}. \quad (32)$$

Imposing the total time constraint, we can rewrite the denominator of the right side of Eq. (32) and get:

$$\frac{\mu}{U} = \frac{\theta}{(\tau - T_w - \sum_{i \in A^R} T_i)}. \quad (33)$$

Recalling that for  $i \in A^R$ ,  $T_i = \sum_j \beta_{ij} X_j$  and noting that if we define  $\eta_j = \sum_{i \in A^R} \frac{\beta_{ij}}{P_j}$  then we can write  $\sum_j \eta_j P_j X_j = \sum_{i \in A^R} T_i$ . Given that the summation in the denominator of the right side of Eq. (33) can be split into two parts, we can write the following:

$$\sum_{i \in A^R} T_i = \sum_{k \in G^F} \eta_k P_k X_k + \sum_{j \in G^R} \eta_j P_j X_j^{\min}. \quad (34)$$

Defining the second term as committed time  $T'_c$ , then Eq. (33) can be re-written as:

$$\frac{\mu}{U} = \frac{\theta}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)}. \quad (35)$$

Dividing Eq. (31) by  $U$  and replacing (35) we obtain the first equation in our system for  $h \in A^F$ :

$$T_h = \frac{\theta_h}{\theta} \left( \tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c \right). \quad (36)$$

Now consider the F.O.C. for  $X_k$  (Eq. (30)) with  $k \in G^F$  (i.e.  $\psi_k = 0$ ), which is:

$$\frac{\varphi_k U}{P_k X_k} - \lambda - \sum_i \kappa_i \frac{\beta_{ik}}{P_k} = 0. \quad (37)$$

Adding Eq. (37) over  $G^F$  and defining  $\varphi \equiv \sum_{k \in G^F} \varphi_k$  we get:

$$\frac{\lambda}{U} = \frac{\varphi - \sum_{k \in G^F} \sum_i \kappa_i \beta_{ik} X_k}{\sum_{k \in G^F} P_k X_k}. \quad (38)$$



We can solve Eq. (28) for  $\kappa_i$ :

$$\kappa_i = \mu - \theta_i U / T_i \quad (39)$$

Replacing (39) and (35) in (38):

$$\frac{\lambda}{U} = \frac{\left( \varphi - \frac{\theta \sum_{k \in G^F} \eta_k P_k X_k}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)} + \sum_{k \in G^F} \sum_i \frac{\theta_i}{T_i} \frac{\beta_{ik}}{P_k} P_k X_k \right)}{\sum_{k \in G^F} P_k X_k}. \quad (40)$$

Rewriting the denominator of Eq. (40) based on the total budget constraint and recalling the definition of committed expenses,  $E_c$ :

$$\frac{\lambda}{U} = \frac{\left( \varphi - \frac{\theta \sum_{k \in G^F} \eta_k P_k X_k}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)} + \sum_{k \in G^F} \sum_i \frac{\theta_i}{T_i} \frac{\beta_{ik}}{P_k} P_k X_k \right)}{(wT_w - E_c)}. \quad (41)$$

Dividing Eq. (37) by  $U$  and replacing (41) we obtain the second equation in our system for  $j \in G^F$ :

$$\frac{\varphi_j}{P_j X_j} - \frac{\left( \varphi - \frac{\theta \sum_{k \in G^F} \eta_k P_k X_k}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)} + \sum_{k \in G^F} \sum_i \frac{\theta_i}{T_i} \frac{\beta_{ik}}{P_k} P_k X_k \right)}{(wT_w - E_c)} - \frac{\varphi \eta_j}{(wT_w - E_c)} + \sum_i \frac{\theta_i}{T_i} \frac{\beta_{ij}}{P_j} P_j X_j = 0 \quad (42)$$

Dividing (29) by  $U$  and replacing (35) and (42) we get the third equation of our system:

$$\frac{\theta_w}{T_w} + \frac{\left( \varphi - \frac{\theta \sum_{k \in G^F} \eta_k P_k X_k}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)} + \sum_{k \in G^F} \sum_i \frac{\theta_i}{T_i} \frac{\beta_{ik}}{P_k} P_k X_k \right)}{(wT_w - E_c)} w - \frac{\theta}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)} = 0. \quad (43)$$

Eqs. (36), (42) and (43) form a system of  $|A^F| + |G^F| + 1$  equations with the same number of unknowns. These unknown decision variables are work time, time assigned to those activities that do not stick to the exogenous minimum, and amount of goods consumed above the corresponding minimum.

Once the system is solved, the rest of the variables (goods and time) can be found as:

$$T_i = \sum_j \beta_{ij} X_j^{\min} \quad \forall i \in A^R \quad (44)$$

$$X_j = X_j^{\min} \quad \forall j \in G^R \quad (45)$$

The value of time as a resource, or value of leisure, can be obtained as:

$$\frac{\mu}{\lambda} = \frac{\theta (wT_w - E_c)}{\left( \varphi - \frac{\theta \sum_{k \in G^F} \eta_k P_k X_k}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)} + \sum_{k \in G^F} \sum_i \frac{\theta_i}{T_i} \frac{\beta_{ik}}{P_k} P_k X_k \right) (\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)}, \quad (46)$$

and then the value of work can be obtained from Eq. (7):

$$\frac{\partial U / \partial T_w}{\lambda} = \frac{\mu}{\lambda} - w. \quad (47)$$

### 3. Model estimation

Considering stochastic error terms ( $u_i, u_i$  and  $v_j$ ) on each F.O.C equation in the model with endogenous consumption of goods and exogenous time allocations, we have<sup>1</sup>:

$$\frac{\tilde{\theta}_w}{T_w} + \frac{\tilde{\varphi} w}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} - \frac{\left( 1 - \frac{\tilde{\varphi} \sum_{h \in A^F} \gamma_h T_h}{(wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h)} + \sum_{h \in A^F} \sum_j \frac{\tilde{\varphi}_j \alpha_{hj} P_j T_h}{P_j X_j} \right)}{(\tau - T_w - T_c)} = u_1 \quad (48)$$

$$\frac{\tilde{\theta}_i}{T_i} - \frac{1 - \frac{\tilde{\varphi} \sum_{h \in A^F} \gamma_h T_h}{wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h} - \sum_{h \in A^F} \sum_j \frac{\tilde{\varphi}_j \alpha_{hj} P_j T_h}{P_j X_j}}{\tau - T_w - T_c} - \frac{\tilde{\varphi} \gamma_i}{wT_w - E_c - \sum_{h \in A^F} \gamma_h T_h} + \sum_j \frac{\tilde{\varphi}_j \alpha_{ij} P_j T_i}{P_j X_j} = u_i \quad \forall i \in A^F \quad (49)$$

<sup>1</sup> In this view, utility is considered deterministic and stochasticity is introduced in the F.O.C conditions. According to this view, not only is the consumer aware of all factors relevant to utility formation, but the analyst observes all of these factors too. However, consumers are assumed to make random mistakes ("errors") in maximizing utility (subject to the many constraints), which gets manifested in the form of stochasticity in the F.O.C conditions. Bhat et al. 2015, in a different context, label such a paradigm as the deterministic utility-random maximization or DU-RM decision postulate. Earlier, Wales and Woodland (1983) also identified this alternative perspective for utility-based models – see footnote 5 in their paper, page 268. As discussed in these earlier works, it can certainly be argued that the DU-RM mechanism is as plausible as the alternative random utility-deterministic maximization (RU-DM) mechanism used more traditionally in microeconomics. Besides, in the current case, the DU-RM mechanism is much easier to work with from a practical viewpoint relative to the RU-DM mechanism.

$$\tilde{\varphi}_j P_j X_j - \tilde{\varphi}_j \left( w T_w - E_c - \sum_{h \in A^F} \gamma_h T_h \right) = v_j \quad \forall j \in G^F. \quad (50)$$

where  $\tilde{\theta}_w = \frac{\theta_w}{\theta}$ ,  $\tilde{\theta}_i = \frac{\theta_i}{\theta}$ ,  $\tilde{\varphi} = \frac{\varphi}{\theta}$  and  $\tilde{\varphi}_j = \frac{\varphi_j}{\theta}$ .

In the case of the model with exogenous minimum consumption of goods and endogenous time allocations, we have:

$$\frac{\tilde{\theta}_w}{T_w} + \frac{\left( \tilde{\varphi} - \frac{\sum_{k \in G^F} \eta_k P_k X_k}{(\tau - T_w - \sum_{j \in G^F} \eta_j P_j X_j - T'_c)} + \sum_{k \in G^F} \sum_i \frac{\tilde{\theta}_i}{T_i} \frac{\beta_{ik}}{P_k} P_k X_k \right)}{(w T_w - E_c)} w - \frac{1}{(\tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k - T'_c)} = u_1 \quad (51)$$

$$T_i - \tilde{\theta}_i \left( \tau - T_w - \sum_{k \in G^F} \eta_k P_k X_k \right) = u_i \quad \forall i \in A^F \quad (52)$$

$$\frac{\tilde{\varphi}_j}{P_j X_j} - \frac{\left( \tilde{\varphi} - \frac{\sum_{k \in G^F} \eta_k P_k X_k}{(\tau - T_w - \sum_{j \in G^F} \eta_j P_j X_j - T'_c)} + \sum_{k \in G^F} \sum_i \frac{\tilde{\theta}_i}{T_i} \frac{\beta_{ik}}{P_k} P_k X_k \right)}{(w T_w - E_c)} - \frac{\tilde{\varphi} \eta_j}{(w T_w - E_c)} + \sum_i \frac{\tilde{\theta}_i}{T_i} \frac{\beta_{ij}}{P_j} P_j X_j = v_j \quad \forall j \in G^F. \quad (53)$$

Due to the existence of the total time constraint (Eq. (3)), only  $L - 1$  time assignment equations can be estimated, where  $L$  is the number of unconstrained activities. Due to the existence of the total budget constraint (Eq. (2)), only  $M - 1$  goods consumption equations can be estimated, where  $M$  is the number of unconstrained goods.

For convenience, we define two new indexes  $l$  and  $m$ , with  $l = 1$  referring to work,  $l = 2, 3, \dots, L$  referring to activities corresponding in set  $A^F$  ( $L$  is the cardinality of set  $A^F$ ), and  $m = 1, 2, \dots, M - 1$  referring to goods in set  $G^F$  ( $M$  is the cardinality of set  $G^F$ ). The left hand of each equation is a function of time assigned to activities,  $f_w(T_w, T_l, X_m)$ ,  $f_l(T_w, T_l, X_m)$ , and consumed goods  $g_m(T_w, T_l, X_m)$ . Then, Eqs. (48) to (50) (for the model with endogenous consumption of goods and exogenous time allocations) or (51) to (53) (for the model with exogenous minimum consumption of goods and endogenous time allocations), can be summarized in:

$$f_l(T_1, \dots, T_{L+1}, X_1, \dots, X_M) = u_l \quad l = 1, 2, \dots, L \quad (54)$$

$$g_m(T_1, \dots, T_{L+1}, X_1, \dots, X_M) = u_m \quad m = 1, 2, \dots, M - 1. \quad (55)$$

Vector  $u = (u_1, u_2, \dots, u_L, v_1, v_2, \dots, v_{M-1})'$  is then assumed to be a realization from a multivariate normal distribution, so that  $u \sim MVN_{L+M-1}(0, \Omega)$  indicates an  $(L + M - 1)$ -variate normal distribution with mean vector of 0 and covariance matrix  $\Omega$ . The probability distribution function of  $u$  is denoted by  $\varphi_{L+M-1}^*(\cdot; 0, \Omega)$ . Then the probability that the individual assigns  $T_1$  to work,  $T_2, \dots, T_L$  to activities in  $A^F$ , and  $X_1, \dots, X_{M-1}$  to goods in  $G^F$  corresponds to:

$$P(T_1, T_2, \dots, T_L, X_1, \dots, X_{M-1}) = |\det(\mathbf{J})| \varphi_{L+M-1}^*(f_1, f_2, \dots, f_L, g_1, g_2, \dots, g_{M-1}), \quad (56)$$

where  $\mathbf{J}$  is the Jacobian of the vector function  $\mathbf{H}(T_1, T_2, \dots, T_L, X_1, \dots, X_{M-1}) = (f_1, f_2, \dots, f_L, g_1, g_2, \dots, g_{M-1})'$  (see Appendix C for the model with endogenous consumption of goods and exogenous time allocations and Appendix D for the model with exogenous minimum consumption of goods and endogenous time allocations). Let  $\omega$  be the diagonal matrix of standard deviations  $\omega_r$  of  $\Omega$ , and let  $\varphi_{L+M-1}(\cdot; \Delta)$  be the multivariate standard normal probability distribution function of dimension  $L+M-1$  and correlation matrix  $\Delta$ . Then,

$$P(T_1, T_2, \dots, T_L, X_1, \dots, X_{M-1}) = |\det(\mathbf{J})| \left( \prod_{r=1}^{L+M-1} \omega_r \right)^{-1} \varphi_{L+M-1}(\omega^{-1} \mathbf{H}; \omega^{-1} \Omega \omega^{-1}) \quad (57)$$

yields the likelihood function:

$$L(\tilde{\theta}_1, \dots, \tilde{\theta}_L, \tilde{\varphi}, \tilde{\varphi}_1, \dots, \tilde{\varphi}_{M-1}, \tilde{\Omega}) = P(T_1, T_2, \dots, T_L, X_1, \dots, X_{M-1}), \quad (58)$$

where  $\tilde{\theta}_1 = \tilde{\theta}_w$ , and  $\tilde{\Omega}$  is the row vectorization of the upper diagonal elements of  $\Omega$ . Due to identification issues, one of the standard deviations of  $\Omega$  has to be fixed to 1. To ensure that the normalized utility parameters  $\tilde{\theta}_2, \dots, \tilde{\theta}_L, \tilde{\varphi}$  and  $\tilde{\varphi}_1, \dots, \tilde{\varphi}_{M-1}$  are positive, we parameterize them as using an exponential function.

## 4. Data

### 4.1. Data description and sample selection

The data used for the analysis is drawn from the LISS panel data. The LISS panel is a representative sample of Dutch individuals who participate in monthly Internet surveys (households that could not otherwise participate are provided with



a computer and Internet connection). The panel is based on a true probability sample of households drawn from the population register, and its first wave was conducted on 2008. A longitudinal survey is fielded in the panel every year, covering a large variety of domains including work, education, income, housing, time use, political views, values and personality.

The LISS panel also includes questionnaires designed by researchers with the purpose of identifying specific behavioral preferences. One of these studies corresponds to a survey on time use and consumption (see [Cherchye et al., 2012](#) for a detailed description). The first wave of these questionnaires was implemented in September 2009, a second wave was conducted in September 2010, and a third in October 2012. In this study, we will focus on the latest wave. The number of individuals available for the analysis is 5463.<sup>2</sup> Respondents reported (1) the time allocated to 13 activities (including work) during the seven days before the survey, and (2) the average monthly expenditure (in euros) in 30 categories, considering as reference the past 12 months. The time use and consumption data are complemented with socio-demographic information drawn from the LISS panel.

The sample used to estimate our model considered individuals who worked at least one hour during the survey week and who reported expenditure in at least one of the expenditure categories. Further, we selected workers who live in one-worker households (i.e., the respondent is the only worker in the household). This last criterion allows assigning all personal and household expenditures to the sole worker in the household, without making assumptions regarding how the household expenditures are shared among income producers. Because time allocation is reported on a weekly basis, the timeframe of our study is a week (including weekends). Monthly expenditures and monthly income are divided by four to obtain weekly expenditures and weekly income, respectively.

The database includes the worker's monthly average gross and net income. For the analysis, only net income is considered. Henceforth, net income is referred as income. Income is disaggregated into salary ( $wT_w$ ) and non-work income ( $I$ ): salary is obtained from working (for an employer or independently) and it is used to compute the wage rate, while non-work income corresponds to the earnings received from pensions, investments, annuities, governmental support, scholarships, tax reimbursement and others non-work related sources.

Several consistency checks were performed to obtain the estimation sample. First, workers with relevant but missing data (such as income and time allocation) were removed from the sample. Second, workers who reported sleeping on average less than 4 h per day were also removed from the sample (accounting for 2.5% of the workers). We hypothesize that individuals who reported sleeping less than 28 h per week may have underestimated their sleeping time and, therefore, misestimated the time assigned to other activities. Third, we removed from the sample those workers who reported extremely high activity durations (for example, some people reported working 168 h per week). Fourth, respondents who spent less than 2 euros per week were removed from the sample, along with those workers whose wage was less than 3 euros/h (the minimum hourly wage in Netherlands was about 8.4 euros in 2012). Finally, we noticed that some workers' expenditure was higher than their income. To correct this inconsistency, we removed from the sample those observations where the difference between expenditure and income was greater than 20% of the worker's income. If the difference between expenditures and income was smaller than 20% of the worker's income, the difference was added to the worker's non-work income ( $I$ ). Therefore, in these last cases, the difference between expenditures and income is zero. After this selection process, the estimation sample included 1193 workers.

#### 4.2. Classification of activities and association of expenditures

The 13 activities available in the original database were grouped into the following 11 activities (three activities –helping parents, helping family members and helping non-family members– were combined into one –assisting friend and family– due to low participation):

- (1) Work: any type of paid work as an employee or as a self-employed worker. The reported time includes overtime hours.
- (2) Commute: travel to and from work, including trips to intermediate stops (such as passing by shops or markets during the way back home).
- (3) Household chores: cleaning, shopping, cooking, gardening, etc.
- (4) Personal care: washing, dressing, eating, visiting the hairdresser, seeing the doctor, etc.
- (5) Education: includes day or evening courses, professional courses, language courses or other course types, doing homework, etc.
- (6) Activities with children: any activity with own children aged less than 16 years, such as washing, dressing, playing, taking child to see doctor, taking child to school/hobby activities, etc.
- (7) Entertainment: in-home and out-of-home recreational activities, such as watching TV, reading, practicing sports, hobbies, computer as hobby, visiting family or friends, going out, walking the dog, cycling, sex, etc.
- (8) Assisting friends and family: assistance to friends and family members (not children). For example: helping with administrative chores, washing, dressing, seeing the doctor, voluntary work, babysitting, etc.
- (9) Administrative chores and family finances.

<sup>2</sup> The third wave of surveys on time use and consumption was administered to 6,874 households. Out of these households, 20.5% did not answer the survey and 2.3% returned incomplete surveys.

**Table 1**  
Descriptive statistics.

Activity	Participation (%)	Duration (hours/week)*				Expenditure (euros/week)*			
		Mean	St. Dev.	Min.	Max.	Mean	St. Dev.	Min.	Max.
Work	100.0	33.4	13.7	1.0	100.0	–	–	–	–
Commute	94.0	4.8	4.8	0.2	60.0	12.8	14.2	0.0	216.0
Household chores	97.8	12.4	9.8	0.3	90.0	5.9	9.8	0.0	107.5
Personal care	100.0	9.1	5.8	0.5	49.0	96.9	66.5	0.0	1005.0
Education	24.7	7.4	9.3	0.2	87.7	1.4	7.4	0.0	125.0
Activities with children	31.2	14.3	11.7	0.5	65.0	17.6	29.1	0.0	166.3
Entertainment	99.8	31.9	16.1	1.0	102.0	38.7	63.1	0.0	725.0
Assisting friends and family	57.6	7.5	7.8	0.2	81.3	–	–	–	–
Administrative chores and family finances	86.6	3.1	3.5	0.2	50.0	–	–	–	–
Sleeping and relaxing	100.0	58.8	11.4	28.0	119.2	–	–	–	–
Going to church and other activities	42.5	11.7	12.5	0.3	71.0	–	–	–	–
Fixed expenditures $c_f$	92.5	–	–	–	–	330.7	179.8	2.4	1316.0
<b>Number of observations</b>	1193								

\* Durations and expenditures are computed only for workers participating in the corresponding activity.

(10) Sleeping and relaxing: sleeping, resting, thinking, meditating, being ill, etc.

(11) Going to church and other activities: going to church, attend funeral/wedding, and any activity not considered above.

To incorporate the novel set of constraints in our model in a way that allows us to make an easy interpretation of the results, we need the expenditure corresponding to the goods allocated to each activity purpose during the survey week. In this way, we are able to relate to each non-work activity  $n$  an associated time ( $T_n$ ) and an associated expense ( $P_n X_n$ ), the latter being the money expenditure related to a composite good  $X_n$  that includes all the goods necessary to perform activity  $n$ . This one-by-one relation between time allocation and good consumption has been a common assumption in the time use microeconomic framework (see for example Becker, 1965, Chiswick, 1967, De Serpa, 1971, Evans, 1972, De Donnea, 1972, Bruzelius, 1979, Juster, 1990, Jara-Díaz, 2003, Jara-Díaz et al., 2008, Konduri et al., 2011, and Jara-Díaz and Astroza, 2013), either for theoretical reasons or because data limitations. In our case, our model formulation does not require this assumption and the LISS data offers a broad range of possibilities regarding time use and good consumption structures, but certainly this assumption makes the interpretation of the results easier.

The LISS panel database contains detailed information about expenditure in 30 distinct categories, but these categories do not directly relate to the activity purposes listed above. Consequently, we needed to associate the expenditures to activities. For this purpose, the expenditure categories were studied in detail to identify those that matched the description of the activities. In addition, we computed the fixed expenditures ( $c_f$ ) as those expenditures not related with any activity purpose. Details about the association procedure and the definition of  $c_f$  can be found in Appendix E. Descriptive statistics are presented in Table 1.

By construction, all individuals in the sample allocate time to work and sleeping/relaxing activities: on average, individuals work 6.6 h per weekday and sleep/relax 8.4 h per day. In addition, all workers spend time in personal care activities (recall that this activity type includes eating and dressing). Most workers allocate some time to commute, entertainment and personal care, while education and activities with children present the lowest participation rates. Regarding expenditure, personal care presents the highest value and it is also the most expenditure-intensive activity (average of 10 euros/h). Although people spend a relatively large amount of money in entertainment activities, these represent only an expenditure rate of 2.2 euros/h, which is considerably lower than the average wage of 18 euros/h.

## 5. Model estimation results

### 5.1. Variable specification and model formulation

To estimate the model, the first step is to classify the 10 non-work activities into the sets defined in our model. This classification is presented in Table 2. Activities with restricted expenses  $G^R$  are subdivided into activities with expenditure restricted at its minimum ( $G_{min}^R$ ) and activities with expenditure restricted to zero ( $G_{zero}^R$ ); in other words,  $G^R = G_{min}^R \cup G_{zero}^R$ . Although this distinction is irrelevant from a model estimation perspective, we believe that it is important to develop an accurate activity classification that recognizes the characteristics of the data used for the analysis.

The set of activities than belong to  $A^R$  (activities restricted in time) and have an associated expense that belongs to  $G^R$  (activities restricted in expenses) comprises the following 6 activities: household chores, personal care, assisting friends and family, administrative chores and family finances, commute and education. That is, individuals spend the smallest amount of time performing these activities, as well as stick to the minimum monetary resources needed to perform the activity. Traditionally personal care and administrative/household chores and personal care have been classified as restricted time activities (see for example Aas, 1982, Bittman and Wajcman, 2000, and Robinson and Godbey, 2010) because of their

**Table 2**  
Classification of activities.

Sets	Restricted expenses $G^R$		Unrestricted expenses $G^F$
	Restricted at minimum $G_{min}^R$	Restricted at zero $G_{zero}^R$	
<b>Restricted activities <math>A^R</math></b>	<ul style="list-style-type: none"> <li>- Household chores</li> <li>- Personal care</li> <li>- Commute</li> <li>- Education</li> </ul>	<ul style="list-style-type: none"> <li>- Assisting friends and family</li> <li>- Administrative chores and family finances</li> </ul>	
<b>Unrestricted activities <math>A^F</math></b>	<ul style="list-style-type: none"> <li>- Activities with children</li> <li>- Entertainment</li> </ul>	<ul style="list-style-type: none"> <li>- Sleeping and relaxing</li> </ul>	<ul style="list-style-type: none"> <li>- Going to church and other activities</li> </ul>

maintenance-oriented nature. People need to take care of their health and hygiene, and manage the household maintenance. These maintenance activities are generally driven by a physical need, but in most cases, individuals do not want to spend more money than necessary to perform such activities (Gronau and Hamermesh, 2006 classified maintenance activities as goods intensive, i.e., individuals really care about the amount of goods they are spending in order to perform these activities; the reader is also referred to Ahn et al., 2004 who observes that individuals generally try to save money in maintenance activities). Similarly, there are other tasks- such as assisting friends and family or family finances- that have to be taken care of. Regarding commute activity, we believe that individuals will assign the minimum necessary because, in general, individuals would rather be doing something else, either at home, at work, or somewhere else, than riding a bus or driving a car. So they will assign the minimum necessary time to commute and, of course, they will not spend more money than necessary no matter which mode of transportation they choose (see Mokhtarian and Chen, 2004 for a review of different studies of travel time and related money expenditures). Finally, we consider that individuals will spend the minimum necessary time in education because classes have a fixed length that usually individuals cannot choose, assignments are mandatory tasks, and extra time of study does not mean extra pay. The expenses associated to commute, household chores, personal care, and education belong to the set  $G_{min}^R$  and the expenses associated to assisting friends and family, and administrative chores and family finances to  $G_{zero}^R$ . There is no activity belonging to  $A^R$  with an associated expense belonging to  $G^F$  (unrestricted regarding expenses, but restricted regarding time). Activities with children, entertainment, and sleeping and relaxing are considered “time unrestricted and expenses restricted” activities, i.e. they belong to  $A^F$  and their associated expense belongs to  $G^R$ . Finally, “going to church and other activities” is the only activity in  $A^F$  and with its associated expense belonging to  $G^F$  (unrestricted in terms of time and expenses). Due to the model derivation it is required that workers allocate some positive expense to activities with associated expenses in set  $G^F$ . However, 57.5% of the sample does not participate in “going to church and other activities”, and there are no expenses associated with this activity in the data. Then, to estimate the model, a small expense (2 euros/week) was appended to “going to church and other activities” for each worker.<sup>3</sup>

As discussed in Section 3, one of the freely chosen activities and one of the freely chosen goods cannot be estimated. The time assigned to “going to church and other activities” and the corresponding expenses are not considered as dependent variables. Consequently, there are four dependent variables in our system (we have a system of four equations): time assigned to work ( $T_w$ ), time assigned to activities with children ( $T_{child}$ ), time assigned to entertainment ( $T_{ent}$ ), and time assigned to sleeping and relaxing ( $T_{sleep}$ ). There are five utility parameters ( $\hat{\theta}_w$ ,  $\hat{\theta}_{child}$ ,  $\hat{\theta}_{ent}$ ,  $\hat{\theta}_{sleep}$  and  $\hat{\varphi}$ ) and nine covariance matrix elements to be estimated ( $\sigma_w$  is fixed to 1 for identification). Since we are considering a one-by-one relation between time assigned to activities and good expenses, we can rewrite the endogenous minimum constraints as  $X_n \geq \alpha_n T_n$ , for the model with endogenous minimum consumption of goods (Eq. (5)), and  $T_n \geq \beta_n X_n$ , for the model with endogenous minimum time allocations (Eq. (26)). For the model with exogenous minimum times and endogenous minimum consumption of goods, the terms  $P_n \alpha_n$  were directly computed from the data by dividing time by the associated expenditure. As we mentioned in Section 2, the estimation does not require the value of  $P_n$  and  $\alpha_n$  independently. With the values of  $P_n \alpha_n$  we can obtain the  $\gamma_n$  values. Similarly, for the model with endogenous minimum times and exogenous minimum consumption of goods, the terms  $\beta_n / P_n$  were directly computed from the data by dividing expenditure by the associated time. With the values of  $\beta_n / P_n$  we can obtain the  $\eta_n$  values.

## 5.2. Estimation results

Tables 3 and 4 present the model estimation results for the two different versions of the model: with exogenous minimum times and endogenous minimum consumption of goods (Table 3), and the model with endogenous minimum times and exogenous minimum consumption of goods (Table 4). The upper section of the tables presents the model parameters and the lower section shows the average computed values of time. For each version of the model we estimated two different error structures: one with identically and independently distributed (iid) error terms and one with a full covariance

<sup>3</sup> A sensitivity analysis was performed to investigate the repercussions of doing so. For values of less than 2 euros, the model could not be estimated. For values equal or greater than 2 euros, the coefficients could be estimated and there was no substantial difference in the results for values between 2 and 25 euros.

**Table 3**

Model parameters; exogenous minimum time, endogenous minimum consumption.

Model parameters				
Coefficients	Model with iid errors		Model with full covariance matrix	
	Estimate	t-stat	Estimate	t-stat
<b>Utility</b>				
$\hat{\theta}_w$	0.095	6.10	0.093	5.04
$\hat{\theta}_{child}$	0.017	7.22	0.014	6.17
$\hat{\theta}_{ent}$	0.078	5.89	0.077	5.19
$\hat{\theta}_{sleep}$	0.686	11.81	0.691	10.23
$\hat{\varphi}$	0.1003	3.09	0.1001	1.11
<b>Covariance matrix</b>				
$\sigma_{work-child}$	0.000	–	0.1736	7.58
$\sigma_{ent-child}$	0.000	–	0.1104	4.59
Log-likelihood	-11,302.8		-11,060.8	
Number of observations	1193		1193	
<b>Average values of time [euros/hr]</b>				
	<b>Estimate</b>	<b>Std. dev.</b>	<b>Estimate</b>	<b>Std. dev.</b>
Leisure	59.57	93.35	59.42	93.11
Work	41.57	95.81	41.42	95.58
Wage	17.99	24.03	17.99	24.03
Ratio leisure – wage		3.31		3.30
Ratio work – wage		2.31		2.30

**Table 4**

Model parameters; endogenous minimum time, exogenous minimum consumption.

Model parameters				
Coefficients	Model with iid errors		Model with full covariance matrix	
	Estimate	t-stat	Estimate	t-stat
<b>Utility</b>				
$\hat{\theta}_w$	0.090	6.00	0.091	4.98
$\hat{\theta}_{child}$	0.016	6.67	0.014	6.05
$\hat{\theta}_{ent}$	0.069	5.90	0.079	5.03
$\hat{\theta}_{sleep}$	0.672	11.02	0.689	9.99
$\hat{\varphi}$	0.100	3.10	0.1002	1.10
<b>Covariance matrix</b>				
$\sigma_{work-child}$	0.000	–	0.1736	7.58
$\sigma_{ent-child}$	0.000	–	0.1104	4.59
Log-likelihood	-11,290.8		-11,024.2	
Number of observations	1193		1193	
<b>Average values of time [euros/hr]</b>				
	<b>Estimate</b>	<b>Std. dev.</b>	<b>Estimate</b>	<b>Std. dev.</b>
Leisure	59.30	93.32	59.32	93.10
Work	42.00	95.78	41.03	95.61
Wage	17.99	24.03	17.99	24.03
Ratio leisure - wage		3.31		3.30
Ratio work - wage		2.31		2.30

matrix (four models were estimated in total). A likelihood ratio test shows that the model incorporating a flexible structure of the error is statistically superior, validating the procedure proposed in this paper (the likelihood ratio test statistic is 484 for the with exogenous minimum times and endogenous minimum consumption of goods and 478 for the model with endogenous minimum times and exogenous minimum consumption of goods, which is much larger than the table chi-squared value with two degrees of freedom at any reasonable level of significance). Tables 3 and 4 show that all the parameters are statistically significant at the 95% level of confidence, but  $\hat{\varphi}$  is associated with a  $p$ -value of only 0.27 in the model with full covariance matrix for both technical constraint formulations.

The value of the parameters do not have a direct interpretation since they are ratios between exponents of the Cobb–Douglas utility function, but the ratio between each pair of  $\hat{\theta}_i$  (including  $\hat{\theta}_w$ ) can be interpreted as the ratio between the elasticities associated with the corresponding variable. Comparing two coefficients can give an idea of the relative

**Table 5**

Values of time for different segments of the population.

	Presence of children in the household				Age			
	No children		At least one child		$\leq 50$ years		$> 50$ years	
	Estimate	Std.dev.	Estimate	Std.dev.	Estimate	Std.dev.	Estimate	Std.dev.
Leisure	69.75	101.83	2.48	3.78	5.74	9.09	93.70	144.51
Work	50.35	103.76	-13.34	17.39	-8.85	15.64	70.61	147.58
Wage	19.40	27.56	15.82	17.00	14.59	13.01	23.09	33.86
Ratio leisure-wage	3.60		0.16		0.39		4.06	
Ratio work-wage	2.60		-0.84		-0.61		3.06	
<b>Location of household</b>					<b>Income level</b>			
	Urban area		Not urban area		Low income		High income	
	Estimate	Std.dev.	Estimate	Std.dev.	Estimate	Std.dev.	Estimate	Std.dev.
Leisure	75.69	117.42	60.62	95.76	17.82	25.11	158.11	188.30
Work	57.37	120.02	42.86	97.45	1.29	32.69	133.72	192.65
Wage	18.32	28.76	17.76	20.02	16.53	20.60	24.38	34.68
Ratio leisure-wage	4.13		3.41		1.08		6.48	
Ratio work-wage	3.13		2.41		0.08		5.48	

importance of each activity/good in terms of utility. In the four models, both technical constraint configurations with both covariance matrix configurations, sleep is the activity with highest impact on utility, following by work, entertainment and, finally, child-care. When the full covariance matrix models were estimated, we found positive correlation between the error term associated with the child-care equation and the error terms associated with the work equation and the entertainment equation. This means that the unobservable factors explaining the child-care equation are also present in the work equation and the entertainment equation. Regarding the values of time, both the value of leisure and the value of work are positive for the four models; consequently, workers considered in the analysis extract pleasure (at the margin) both from working and undertaking leisure activities. For the model with endogenous minimum consumption of goods and exogenous time allocations, the value of time of leisure is estimated as 59.6 euros/h and the value of time of work as 41.6 euros/h. For the model with exogenous minimum consumption of goods and endogenous time allocations the results are slightly different; the value of time of leisure is estimated as 59.3 euros/h and the value of time of work as 42.0 euros/h. If all the activities belonging to  $A^R$  would have an associated monetary expenditure belonging to  $G^R$ , then we can write for those activities the endogenous minimum constraints as  $X_n = \alpha_n T_n$ , for the model with endogenous minimum consumption of goods, and  $T_n = \beta_n X_n$ , for the model with endogenous minimum time allocations. If in addition, all the activities belonging to  $A^F$  would have an associated monetary expenditure belonging to  $G^F$ , i.e. the freely chosen activities/goods do not impact the technological constraints, then we can simply write the relation  $\alpha_n = 1/\beta_n$  and both models (with both technical constraint formulations) would be equivalent. However, in our specification some of the activities (activities with children, entertainment and sleeping and relaxing) have associated restricted expenses but are unrestricted about time. This breaks the symmetry between  $A^F$  and  $G^F$  and provokes differences in the value of the estimated Cobb–Douglas coefficients and, consequently, differences in terms of the value of time.

As mentioned in the previous sections, the novelty of the proposed model is the introduction of a link between minimum consumption and time. To assess the contribution of our approach, we estimated a model that does not incorporate this link, as developed by Jara-Díaz et al. (2008), and computed the corresponding values of time. The model estimation results can be found in Appendix F. The resulting values of leisure and work are 122.8 euros/h and 104.8 euros/h, respectively. If we compare the value of leisure and work among the models, we can identify a clear difference: the model without the link overestimates the values of time. In other words, when omitting the relation between consumption and time, the model cannot correctly capture the individual valuation of time. Intuitively, since we are considering in one version of the current model that some of the freely chosen activities (those that individuals assign more time than the minimum necessary) are imposing lower bounds (or minimum requirements) to goods consumption and- consequently- expenses, leisure time has a “cost” in this formulation. This differs from previous models- without our link between goods and activities- that only consider a pure cost-free leisure time. In the other case, when we are considering that some of the goods are imposing lower bounds to time assigned to those activities that are restricted to the minimum, then the consumption of goods is directly related to the restricted time and consequently, to the time available for leisure. Since the model without the link does not consider this ‘time cost’ of the consumption of goods, the value of leisure time is overestimated.

Finally, we explored several segments of the population to identify differences in the valuation of time. The segmentation was made using socio-demographic data, including age, gender, income, education, location of the household (urban vs. rural), presence of children in the household and whether the worker had a partner (see Jara-Díaz et al., 2013). Further, we explored combinations of the previous segments (for example, we compared the values of time of females with children and females without children). Table 5 reports those segments that showed statistical differences in their valuations of time

using the model with exogenous minimum times and endogenous minimum consumption of goods only, as both versions yield very similar results.

The value of leisure is positive (as expected) for all segments and the value of work is negative for some the segments as follows. Workers who have children present a negative value of work time, indicating that they do not extract pleasure from work at the margin; on the other hand, workers without children enjoy their work at the margin. This result could be related with the financial freedom perceived by workers who do not have to economically support children: they can choose a more satisfying job than workers who need to provide for their family. According to several earlier studies (see, for example, Kim et al., 2005, Uunk et al., 2005, Baxter et al., 2007, and Compton and Pollak, 2014), parents – especially women – perceive less job autonomy (the freedom to decide how they do their work) and more pressure regarding job location selection than individuals without children. Another explanation is that parents prefer to spend time out of work to share it with their children (Sayer et al., 2004).

Young workers (aged less or equal than 50 years) have a negative value of work, while older workers (aged more than 50 years) have a positive one. It is possible that young workers, compared to old workers, have more debt or commitments (college debt, mortgage) that, to some extent, force them to choose unsatisfying jobs.<sup>4</sup> Also, earlier studies have shown that older workers generally have more positive job attitudes (such as overall job satisfaction, satisfaction with work itself, satisfaction with pay, job involvement, emotional exhaustion, or satisfaction with coworkers) than younger workers (see Rhodes, 1983, Carstensen, 1992, Mather and Johnson, 2000, and Ng and Feldman, 2010). Regarding the location of the household, Table 4 shows that the valuation of time is higher for workers living in urban areas (although wages are statistically the same). A plausible explanation of this result is that workers in urban areas can participate in many activities that are not feasible in rural areas, such as attending cultural activities, shopping and eating out. Then, due to increased accessibility, these workers perceive their times as more valuable than workers whose houses are located in rural areas (Farrington and Farrington, 2005). Finally, income is a relevant determinant of value of time. Our results show that low income workers (monthly income less or equal to 4000 euros) have a lower valuation of time than high income workers (monthly income greater than 4000 euros).

## 6. Conclusions

We have developed a model explicitly introducing a piece that was missing in previous models of time use, namely a relation between goods consumed and the time assigned to activities that use it. Although a closed solution for activities and work time could not be found, this indeed improves over the previous most advanced microeconomic formulations because minimum levels of consumption or time assignment to activities become endogenous. However, we have generated a system of equations where the decision variables are work, those activities that are assigned more than the minimum and those goods that are consumed more than needed. From this system, the parameters of the implicit equations can be estimated using maximum likelihood techniques without assuming independence of the error terms. We further discuss identifiability issues and explicitly compute the Jacobian resulting from the likelihood multivariate integral.

Our microeconomic framework is applied to a Dutch weekly time use and consumption database. To our knowledge, this is one the few surveys in the world that includes both time allocation and good consumption information. Using the estimated model parameters, we computed the values of time (value of leisure and value of work), which are considerable higher than the wage. A comparison of these estimates with those from a model that does not include the additional constraint shows substantial differences: the values of time for the model without the link are about twice the values of time of our proposed model, showing the importance of correctly introducing relations between time allocation and good consumption in the modeling framework. This empirical result is particularly relevant from a policy standpoint, as a miscalculation of the value of time can lead to erroneous computation of the benefits of public investment projects. In addition, value of time estimations were performed on different segments of the population. Significant differences in the valuation of time were observed when segmenting by income, age, location of the household (urban vs. rural) and presence of children in the household, providing interesting insights regarding Dutch workers' preferences and lifestyles.

But a better understanding of the social elements behind the perception, valuation and use of time is not the only practical use of the improved models. Forecasting changes in time use after changes in technology as faster transit services or improved ways to do errands (e.g. teleshopping), is also feasible with the estimated models because the equations systems could be used to simulate the impact by simply varying the (exogenous) parameters affected: min goods or min times levels, or the alpha-price. So, beyond a better understanding of the impact of technical change on consumption and time use at a micro level, knowing the new time assignments and the value gained by the individuals through the extra leisure is of great importance from a practical viewpoint.

By way of future extensions, we are working on a discrete choice framework for the decision to assign time to activities. Participation choice could be very important to address the censored nature of time allocation, and could allow a non-arbitrary mechanism for observations with zero values for the dependent variables. This additional discrete dimension in our model would certainly be interesting for future research.

<sup>4</sup> We hypothesized that young workers were more likely to have children living with them than old workers. A hypothesis test concluded that there is no significant correlation between age and presence of children in the household, rejecting our initial hypothesis.



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## Appendix A. Derivation of first order conditions for the model with exogenous minimum times and endogenous minimum consumption of goods

The Lagrangian function is given by:

$$\begin{aligned} L(\mathbf{T}, \mathbf{X}) = & \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j X_j^{\varphi_j} + \lambda (wT_w + I - \sum_i P_i X_i - c_f) + \mu (\tau - T_w - \sum_i T_i) \\ & + \sum_i \kappa_i (T_i - T_i^{\min}) + \sum_j \psi_j (X_j - \sum_i \alpha_{ij} T_i) \end{aligned} \quad (\text{A.1})$$

Then the partial derivative of the lagrangian respect to each decision variable is:

$$\frac{\partial L}{\partial T_i} = \frac{\theta_i U}{T_i} + \kappa_i - \mu - \sum_j \psi_j \alpha_{ij} \quad (\text{A.2})$$

$$\frac{\partial L}{\partial T_w} = \frac{\theta_w U}{T_w} + \lambda w - \mu \quad (\text{A.3})$$

$$\frac{\partial L}{\partial X_j} = \frac{\varphi_j U}{X_j} - \lambda P_j + \psi_j. \quad (\text{A.4})$$

## Appendix B. Derivation of first order conditions for the model with endogenous minimum times and exogenous minimum consumption of goods

The Lagrangian function is given by:

$$\begin{aligned} L(\mathbf{T}, \mathbf{X}) = & \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j X_j^{\varphi_j} + \lambda (wT_w + I - \sum_i P_i X_i - c_f) + \mu (\tau - T_w - \sum_i T_i) \\ & + \sum_i \kappa_i (T_i - \sum_j \alpha_{ij} X_j) + \sum_j \psi_j (X_j - X_j^{\min}) \end{aligned} \quad (\text{B.1})$$

Then the partial derivative of the Lagrangian respect to each decision variable is:

$$\frac{\partial L}{\partial T_i} = \frac{\theta_i U}{T_i} + \kappa_i - \mu \quad (\text{B.2})$$

$$\frac{\partial L}{\partial T_w} = \frac{\theta_w U}{T_w} + \lambda w - \mu \quad (\text{B.3})$$

$$\frac{\partial L}{\partial X_j} = \frac{\varphi_j U}{X_j} - \lambda P_j - \sum_i \kappa_i \alpha_{ij} + \psi_j. \quad (\text{B.4})$$

## Appendix C. Computation of the elements of the Jacobian – Endogenous minimum consumption of goods and exogenous minimum times

The elements of the Jacobian are given by:

$$J_{lh} = \begin{cases} \frac{\partial f_l(T_1, \dots, T_{L+1}, X_1, \dots, X_M)}{\partial T_w} & \text{for } h = 1 \\ \frac{\partial f_l(T_1, \dots, T_{L+1}, X_1, \dots, X_M)}{\partial T_h} & \text{for } 2 \leq h \leq L \\ \frac{\partial f_l(T_1, \dots, T_{L+1}, X_1, \dots, X_M)}{\partial X_h} & \text{for } L+1 \leq h \leq L+M-1 \end{cases} \quad (C.1)$$

where function  $f_l$  is defined in Eq. (21) for  $l = 1$  (work), in Eq. (20) for  $l = 2, 3, \dots, L$  (activities in  $A^F$ ), and (14) for  $l = L+1, \dots, L+M-1$  (activities in  $G^F$ ). Let

$$C = \left[ \frac{1}{wT_w - E_c - \sum_{i \in A^F} \gamma_i T_i} \right]^2 \tilde{\varphi} \quad (C.2)$$

$$D = \left[ \frac{1}{\tau - T_w - T_c} \right]^2 \quad (C.3)$$

Then, the  $lh$ th element of the Jacobian is:

$$J_{11} = -\tilde{\theta}_w T_w^{-2} - w^2 C - D \quad (C.4)$$

$$J_{1h} = \begin{cases} wCP_h \alpha_h - D & \text{for } 2 \leq h \leq L \\ 0 & \text{for } L+1 \leq h \leq L+M-1 \end{cases} \quad (C.5)$$

$$J_{l1} = \begin{cases} -D + P_l \alpha_l Cw & \text{for } 2 \leq h \leq L_1 + 1 \\ -D & \text{for } L_1 + 2 \leq h \leq L + 1 \end{cases} \quad (C.6)$$

$$J_{lh} = \begin{cases} -\frac{\tilde{\theta}_l}{T_l^2} z_{lh} - D - CP_h \alpha_h P_l \alpha_l & \text{for } 2 \leq l, h \leq L_1 + 1 \\ 0 & \text{for } 2 \leq l \leq L_1 + 1 \text{ and } L_1 + 2 \leq h \leq L + 1 \\ -D & \text{for } L_1 + 2 \leq l \leq L + 2 \text{ and } 2 \leq h \leq L_1 + 1 \\ -\frac{\tilde{\theta}_l}{T_l^2} z_{lh} & \text{for } L_1 + 2 \leq l, h \leq L + 1 \end{cases} \quad (C.7)$$

where  $z_{lh} = 1$  if  $l = h$  and  $z_{lh} = 0$  if  $l \neq h$ . There is no closed-form structure for the determinant of the Jacobian.

#### Appendix D. Computation of the elements of the Jacobian – Exogenous minimum consumption of goods and endogenous minimum times

The elements of the Jacobian are given by:

$$J_{lh} = \begin{cases} \frac{\partial f_l(T_1, \dots, T_{L+1}, X_1, \dots, X_M)}{\partial T_w} & \text{for } h = 1 \\ \frac{\partial f_l(T_1, \dots, T_{L+1}, X_1, \dots, X_M)}{\partial T_h} & \text{for } 2 \leq h \leq L \\ \frac{\partial f_l(T_1, \dots, T_{L+1}, X_1, \dots, X_M)}{\partial X_h} & \text{for } L+1 \leq h \leq L+M-1 \end{cases} \quad (D.1)$$

where function  $f_l$  is defined in Eq. (29) for  $l = 1$  (work), in Eq. (30) for  $l = 2, 3, \dots, L_1 + 1$  (activities in  $A^F$ ), and (31) for  $l = L_1 + 2, \dots, L + 1$  (activities in  $G^F$ ). Let

$$C = \left[ \frac{1}{wT_w - E_c - \sum_{i \in A^F} \gamma_i T_i} \right]^2 \tilde{\varphi} \quad (D.2)$$

$$D = \left[ \frac{1}{\tau - T_w - T_c} \right]^2 \quad (D.3)$$

Then, the  $lh$ th element of the Jacobian is:

$$J_{11} = -\tilde{\theta}_w T_w^{-2} - w^2 C - D \quad (D.4)$$

$$J_{1h} = \begin{cases} wCP_h \alpha_h - D & \text{for } 2 \leq h \leq L_1 + 1 \\ 0 & \text{for } L_1 + 2 \leq h \leq L + 1 \end{cases} \quad (D.5)$$

$$J_{l1} = \begin{cases} -D + P_l \alpha_l Cw & \text{for } 2 \leq h \leq L_1 + 1 \\ -D & \text{for } L_1 + 2 \leq h \leq L + 1 \end{cases} \quad (D.6)$$

$$J_{lh} = \begin{cases} -\frac{\hat{\theta}_l}{T_l^2} z_{lh} - D - CP_h \alpha_h P_l \alpha_l & \text{for } 2 \leq l, h \leq L_1 + 1 \\ 0 & \text{for } 2 \leq l \leq L_1 + 1 \text{ and } L_1 + 2 \leq h \leq L + 1 \\ -D & \text{for } L_1 + 2 \leq l \leq L + 2 \text{ and } 2 \leq h \leq L_1 + 1 \\ -\frac{\hat{\theta}_l}{T_l^2} z_{lh} & \text{for } L_1 + 2 \leq l, h \leq L + 1 \end{cases} \quad (D.7)$$

where  $z_{lh} = 1$  if  $l = h$  and  $z_{lh} = 0$  if  $l \neq h$ . There is no closed-form structure for the determinant of the Jacobian.

## Appendix E. Association of expenditures to activities

Activity	Expenditure category considered
Commute	<ul style="list-style-type: none"> <li>Average weekly household expenditure transportation, multiplied by 0.36.</li> </ul> <p><i>Assumption:</i> According to a recent study in the Netherlands, about 18% of all trips are trips to work (Bohte and Maat, 2009). Then, trips to and from work account for about 36% of all trips.</p>
Household chores	<ul style="list-style-type: none"> <li>Average weekly household expenditure in cleaning the house or maintaining the garden, divided by the number of adults in the household.</li> </ul> <p><i>(Assumption: all adults in the household equally participate in household chores).</i></p>
Personal care	<ul style="list-style-type: none"> <li>Average weekly personal expenditure in eating at home.</li> </ul> <p><i>Assumption:</i> Some respondents did not declare their expenditure in this category, but they reported the household expenditure in eating at home (for all household members). Then, for those respondents with missing data, this expenditure was computed as the household expenditure divided by the household size. The assumption is that all household members consume the same amount of food. This was validated by computing the proportion of personal expenditure in eating at home, compared to the total household expenditure which, in average, was consistent with this assumption.</p> <ul style="list-style-type: none"> <li>Average weekly personal expenditure in food and drinks outside the house.</li> <li>Average weekly personal expenditure in personal care products and services.</li> <li>Average weekly personal expenditure in medical care and health costs not covered by insurance.</li> </ul>
Education	<ul style="list-style-type: none"> <li>Average weekly personal expenditure in (further) schooling.</li> </ul>
Activities with children	<ul style="list-style-type: none"> <li>Expenditure per week for children living at home in: food and drinks outside the house, cigarettes and other tobacco products, clothing, personal care products and services, medical care and health costs not covered by insurance, leisure time expenditure, (further) schooling, donations and gifts, other expenditures.</li> </ul> <p><i>Assumption:</i> all children-related expenditure is associated with the time spent with them. This is not necessarily true: a parent can buy food and drinks outside the house for the children, but not spend time while the children eat. Or he/she can purchase a movie ticket and do not go with the children to the cinema. However, the survey does not provide information that allows us to identify the relationship between expenditures and activities with children, and we decided to consider all expenditures related with children.</p>
Entertainment	<ul style="list-style-type: none"> <li>Average weekly personal expenditure in leisure time expenditure.</li> <li>Average weekly household expenditure in daytrips and holidays with the whole family or part of the family.</li> </ul>
Assisting friends and family	No expenditure.
Administrative chores and family finances	No expenditure.
Sleeping and relaxing	No expenditure.
Going to church and other activities	No associated expenditure.

In addition, we constructed the fixed expenses  $c_f$  considering the following expenditure categories:

- Average weekly household expenditure in mortgage and rent.
- Average weekly household expenditure in general utilities and insurances.
- Average weekly household expenditure in children's daycare.
- Average weekly household expenditure in alimony and financial support for children not (or no longer) living at home.
- Average weekly household expenditure in debts and loans.
- Average weekly household expenditure in other household expenditure.
- Average weekly household expenditure transportation, multiplied by 0.64 (corresponds to the expenditure on transportation for other household members and non-work travel).
- Average weekly household expenditure in eating at home, minus average weekly household expenditure in eating at home (corresponds to the expenditure in eating at-home for other household members).

## Appendix F. Model with no link between consumption and time allocation

Coefficient	Model with iid errors		Model with full covariance matrix	
	Estimate	t-stat	Estimate	t-stat
<b>Utility</b>				
$\alpha$	0.489	44.82	0.488	366.69
$\beta$	0.105	7.33	0.104	66.86
$\gamma_{child}$	0.036	1.57	0.036	16.98
$\gamma_{ent}$	0.254	10.93	0.255	73.70
$\gamma_{sleep}$	0.460	18.73	0.461	162.31
$\gamma_{other}$	0.040	1.77	0.040	17.90
<b>Covariance matrix</b>				
$\sigma_{work}$	100.000	–	106.101	24.42
$\sigma_{work-child}$	0.000	–	9.824	3.53
$\sigma_{work-ent}$	100.000	–	85.707	24.42
$\sigma_{work-sleep}$	0.000	–	53.234	11.28
$\sigma_{work-other}$	0.000	–	-47.331	-11.18
$\sigma_{child}$	100.000	–	223.192	24.42
$\sigma_{child-ent}$	0.000	–	31.106	8.83
$\sigma_{child-sleep}$	0.000	–	-19.435	-6.24
$\sigma_{child-other}$	0.000	–	-62.645	-11.90
$\sigma_{ent}$	100.000	–	130.307	24.40
$\sigma_{ent-sleep}$	0.000	–	0.000	–
$\sigma_{ent-other}$	0.000	–	0.000	–
$\sigma_{sleep}$	0.000	–	0.000	–
$\sigma_{sleep-other}$	0.000	–	0.000	–
$\sigma_{other}$	100.000	–	98.168	4.02
Log-likelihood		-32,989.7		-22,152.3
Number of observations		1193		1193
	<b>Estimate</b>	<b>Std. dev.</b>	<b>Estimate</b>	<b>Std. dev.</b>
Leisure	132.67	234.16	122.80	215.23
Work	114.68	213.00	104.81	194.08
Wage	17.99	24.03	17.99	24.03

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